Handbook of Global Analysis
Demeter Krupka and David Saunders



# De Rham cohomology[1]

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### Introduction

A (co)homology theory is a functor from a subcategory of the category of topological spaces (e. g. the category of manifolds, the category of CW-complexes, etc.) to an algebraic category (e. g. the category of Abelian groups, the category of rings, etc) satisfying additional axioms. For the category of manifolds it is natural to have a cohomology theory which is constructed via the differentiable structure. The de Rham cohomology theory is a classical cohomology theory of this type. For the most important classes of differentiable manifolds, de Rham cohomology coincides with cohomology constructed in a pure topological way, and therefore gives us a possibility to establish relation between the invariants of differential geometric structures on a manifold and the topological invariants of this manifold. This fact results in numerous applications of de Rham cohomology: various characteristic classes, de Rham-like complexes associated to differential structures, etc.

The aim of the present paper is to give a brief introduction to the de Rham cohomology theory and to expose some relevant results in differential geometry. We do not give proofs, however we provide the reader with references to literature where he can find detailed exposition including proofs of results formulated here.

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